RESONANCE EFFECTS DURING SCATTERING AND ABSORPTION OF RADIO WAVES IN MODELS OF TRAILS OF FAST MOVING BODIES IN THE IONOSPHERE

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ABSTRACT. A mathematical analysis is made of resonance effects in meteor trails during radio wave scattering and absorption.

It is known [1] that the region of a trail left by a moving body in the ionosphere can have natural oscillations and that the existence of a wave number whose squared reciprocal is close to zero is analogous to the emergence of a "bound state" with a near-zero energy level in quantum mechanics. Therefore, resonance scattering of TM waves transmitted from the earth can take place in a trail with the shape of an infinite circular cylinder, the electron concentration in which depends only on the radius. /73

The trail may not have the true symmetry of a circular cylinder under real conditions, and generally speaking is flattened by the earth's magnetic field degenerating into a strip, and then into a flat layer. We can, in order to take into consideration this effect on the scattering cross section radio wave frequency ratio, consider the problem of resonance scattering on an elliptical cylinder of infinite length in which the semiminor axis, b, is less than a wavelength, while the ratio between the wavelength and the semimajor axis, a, is arbitrary.

The scattering cross section in this cylinder, as we know [3], has the following form in the case of normal incidence (parallel to the semiminor axis) of a plane TM wave

$$\sigma = \frac{8\pi}{\varkappa} \sum_{m=0}^{\infty} \frac{Se^2_m(h,0)}{N_m(h)} \sin^2\delta_m, \qquad (1)$$

where

$Se_m(h,\cos\varphi)$ is the Mathieu function,

$$N_m(h) = \int_0^{2\pi} Se^2_m(h,\cos\varphi)\,d\varphi, \qquad /74$$

$h = \varkappa a$, δ_m are phase shifts.

Assuming that for given length a there exists a single eigenvalue, $\varkappa_0 = -i\varkappa$, close to zero, which is a nonessential singularity in the scattering

amplitude, it is not difficult to find the value of the phase, δ_m, close to resonance from the results [1]

$$\delta_m \simeq -\operatorname{arctg} \frac{x}{x_0}. \tag{2}$$

Let us limit ourselves to establishing the obvious dependence of the scattering cross section on h for a band greatly elongated along the OX axis (along a), when the contribution of the ends distant from the center to scattering can be ignored. The basic contribution to the value of the Mathieu functions $Se_m(h,\cos\varphi)$ is made by angles close to $< 1 = \pm \pi/2$ [3] for large values of the parameter $h = \varkappa a$. On the other hand, the Se_m (h, z) functions are proportional to radial Mathieu functions of the first kind, Ie_m(h, z) [3], which, for arbitrary h, can be represented in the form of an expansion in Bessel functions

$$Ie_m(h,z) = \sum_{n=0}^{\infty} B_n(h,m) I_n(hz). \tag{3}$$

The influence of the ends of the band obviously will be nonexistent if, for given h and $z \to 0$, one is limited to just the first term in Eq. (3). Then, substituting Eqs. (2) and (3) into Eq. (1) the approximation result is

$$\sigma = \frac{2\pi^2 \varkappa h}{x^2 + x_0^2} \left(\int_0^{x_0} \frac{\sin^2 x}{x^2} dx \right)^{-1}. \tag{4}$$

where

$$x_0 = \frac{\pi^2 h}{2}.$$

Eq. (4) should be proportional to the transmission coefficient, T, for a plane layer when $h \to \infty$. Actually, Eq. (4) yields for $h \to \infty$

$$\sigma = \frac{2\pi^2 \varkappa h 8}{x^2 + x_0^2} \left(\int_0^{\infty} \frac{\sin^2 x}{x^2} dx \right)^{-1} = \frac{4\pi a x^2}{x^2 + x_0^2}. \tag{5}$$ /75

Hence

$$\left.\begin{aligned} T &= \frac{\sigma}{4\pi a} = \frac{x^2}{x^2 + x_0^2} \\ R &= 1 - T = \frac{x_0^2}{x^2 + x_0^2} \end{aligned}\right\}, \tag{6}$$

which coincides with the resonance coefficients of transmission and reflection from a plane layer [1].

Eq. (4) should be valid as well for $h \to 0$ (the contribution of ends, and maximum deviations of the cross section from a circle are obviously not significant in this case), when the integral in the denominator of Eq. (4) equals $\pi^2 h/2$

$$\sigma \approx \frac{4\varkappa}{\varkappa^2+\varkappa_0^2}. \tag{7}$$

The last result can be generalized for the case of an extended region of finite length, L, if the connection between the amplitudes of the scattering in a three-dimensional region and a two-dimensional one is used [2]

$$\sigma \doteq \frac{4\varkappa L}{\varkappa^2+\varkappa_0^2(L)}, \tag{8}$$

where the eigenvalue $\varkappa_0(L)$, close to zero, exists only for

$$L_{cr} \geqslant \frac{1}{\varkappa_0(\infty)}.$$

It is interesting to note that Eq. (8) is the limit case for resonance scattering of scalar waves by a prolate spheroid, if one excludes the influence of its ends on the scattering

$$\sigma \approx \frac{4\pi h}{\varkappa^2+\varkappa_0^2}\left(\int_0^h \frac{\sin^2 x}{x^2}\,dx\right)^{-1}, \tag{9}$$

where

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$h = \varkappa c$, and 2c is the interfocal distance for the spheroid.

Taking the limit as $h \to \infty$ in expression (9), we get (8), and as $h \to 0$ we get Wigner's result in quantum mechanics

$$\sigma \approx \frac{4\pi}{\varkappa^2+\varkappa_0^2}. \tag{10}$$

The results obtained are true for a nonabsorbing plasma when the degree of perturbation, U, is real, something that is valid for example, for perturbations caused by a moving body in the upper layers of the ionosphere,

by interplanetary gas, or by outer space, without including the effect of a permanent magnetic field. Under these conditions the length of the free particle path, l, is long compared to the dimensions of the moving body, so the influence of collisions on the perturbation of particle concentration is insignificant in a large region in the vicinity of the body. Collisions between particles begin to have an effect only at long distances from the body (of the order of the free path length), as well as in the lower layers of the ionosphere, where the free path length is not as long. The influence of collisions between particles therefore must be taken into consideration in these regions; that is, the perturbation of a complex magnitude must be taken into consideration. The complex magnitude of the perturbation implies that at distances of the order of the path length the trail region, based on its scattering properties, is analogous to a complex potential well, in which absorption is significant as well as scattering. Absorption can be the principal effect if the energy of the scattered particle vanishes.

Let us, in order to explain the qualitative features of complex perturbation on the process of scattering and absorbing radio waves in a trail, consider the problem of the scattering of a TM wave by a low-capture cylindrical inhomogeneity. The small imaginary part of the perturbation makes the phase angles, δ_m, contained in the scattering amplitude, complex

$$\tilde{\delta}_m = \delta_m + i\alpha_m , \tag{11}$$

where

$$\mathrm{Re}\tilde{\delta}_m \gg \mathrm{I}_m\tilde{\delta}_m ,$$

α_m is proportional to the frequency of collisions between particles.

Based on the arguments, the work in reference [1] can only be limited by a wave with m = 0. The scattering and absorption cross sections then can be calculated using the formulas [3] /77

$$\sigma_s = \frac{1}{x}\left|1 - e^{2i\tilde{\delta}_0}\right|^2 ,$$

$$\sigma_n = \frac{1}{x}\left(1 - \left|e^{2i\tilde{\delta}_0}\right|^2\right) \approx \frac{4|\alpha_0|}{x} .$$

The phase angle in the resonance region is approximately

$$\delta^p_0 \simeq -\operatorname{arctg}\frac{x}{x_0},$$

therefore

$$\sigma^p_s \simeq \frac{4x}{x^2+x^2_0}(1-2\,|\,\alpha^p_0\,|\,),$$

$$\sigma^p_n = \frac{4\,|\,\alpha^p_0\,|}{x}. \tag{12}$$

Far from the resonance $\sigma_0 = \lambda_0$, $\alpha_0 = \alpha^0_0$, therefore

$$\sigma^0_s = \frac{4}{x}(1-2\,|\,\alpha^0_0\,|\,)\sin^2\lambda_0,$$

$$\sigma^0_n = \frac{4\,|\,\alpha^0_0\,|}{x}, \tag{13}$$

and $\lambda_0 \sim (\varkappa a)^2$, such that $\sigma^0_s \sim \varkappa^3 a^4$, where a is the effective radius of the cylindrical inhomogeneity.

Comparing Eqs. (13) and (12), we see that when $\varkappa \to 0$ the absorption becomes the dominant effect if $\varkappa$ is not in the resonance region. The ratio of the scattering cross section far from the resonance to the cross section is of the order of $\varkappa^4 a^4/\alpha^0_0$, such that $\sigma^0_s < \sigma^0_n$ if $\alpha^0_0 > \varkappa^4 a^4$.

Conversely, near the resonance, when $\varkappa = \varkappa_0$

$$\sigma^p_{s\,max} = \frac{2}{x_0}(1-2\,|\,\alpha^p_{0\,max}\,|),$$

$$\frac{\sigma^p_{s\,max}}{\sigma^p_{n\,max}} \simeq \frac{1}{2\,|\,\alpha^p_{0\,max}}, \tag{14}$$

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and since $\alpha^p_{0max} \ll 1$, then $\sigma^p_s \gg \sigma^p_n$; that is, in this case resonance scattering can be the dominant effect.

The value of the imaginary part of the phase angle contained in the absorption and scattering cross sections can be determined by the method of analytical continuation from the region of actual phase values to a complex region.

For simplicity let us limit ourselves to a model independent of perturbation coordinates with a sharp boundary such that in the trail region

$$\tilde{U}_0 = U_0 + i\gamma$$

that is, the permittivity in the region of perturbing activity is complex

$$\tilde{\varepsilon}=\varepsilon_1+i\varepsilon_2=\tilde{n^2}=(n_1+in_2)^2\simeq n^2{}_1+2in_1n_2,$$

hence

$$\varepsilon_1=n^2{}_1,\ n_2=\frac{\varepsilon_2}{2\sqrt{\varepsilon_1}}.$$

Expanding $\tilde{\delta}_0$ in a Taylor series, we obtain

$$\alpha_0=n_2\frac{\partial\delta_0}{\partial n_1}=\frac{\partial\delta_0}{\partial\varepsilon_1}I_m\tilde{\varepsilon}.$$

The phase $\delta_0{}^0$ (the "potential scattering" phase) is equal to [2]

$$\delta^0{}_0=\frac{\pi a^2\varkappa^2}{4}\left|\frac{\varepsilon_1}{\varepsilon_0}-1\right|,$$

where

ε_0 is the permittivity outside the trail. Then

$$\sigma^0{}_n=\frac{\pi a^2\varkappa^2}{x}I_m\tilde{\varepsilon},$$

$$\sigma^0{}_s=\frac{\pi^2a^4\varkappa^3}{4}\left|\frac{\varepsilon_1}{\varepsilon_0}-1\right|^2.$$

Hence, it will be seen

$$\sigma^0{}_n>\sigma^0{}_s, \qquad (15)$$ /79

if

$$I_m\tilde{\varepsilon}>\frac{\pi a^2\varkappa^2}{4}\varepsilon_1.$$

Inequality (15) is a consequence of the fact that when $\lambda_0\to\infty$ (λ_0 is the wavelength in free space) $\sigma^0{}_n\sim a^2/\lambda_0$ at the same time that $\sigma^0{}_s\sim a^4/\lambda^3{}_0$ [2]. Near resonance, $\delta^p{}_0\simeq$ -arc tg$(\varkappa/\varkappa_0)$, where

$$\varkappa_0=\frac{2}{a}\exp(-2/\varkappa^2a^2\varepsilon_1),$$

therefore

$$\alpha^2{}_0=\frac{\varkappa\varkappa_0\ln\frac{2}{\varkappa_0a}}{\varkappa^2+\varkappa^2{}_0}\cdot\frac{I_m\tilde{\varepsilon}}{Re\tilde{\varepsilon}},$$

$$\sigma^p{}_n=\frac{4\varkappa_0}{\varkappa^2+\varkappa^2{}_0}\ln\frac{2}{\varkappa_0a}\cdot\frac{Im\tilde{\varepsilon}}{Re\tilde{\varepsilon}},$$

$$\sigma^p{}_s\simeq\frac{4\varkappa}{\varkappa^2+\varkappa^2{}_0}.$$

Hence, we obtain the fact that the ratio of the maximum resonance absorption and scattering cross section is equal to $(\mathrm{Im}\tilde{\varepsilon}/\mathrm{Re}\tilde{\varepsilon})\ \ln^2/\varkappa_0 a$. And since $\mathrm{Im}\tilde{\varepsilon} \ll \mathrm{Re}\tilde{\varepsilon}$, then a given, $\varkappa_0 \sigma^p{}_{n\ \max} \ll \sigma^p{}_{s\ \max}$ if $(\mathrm{Im}\tilde{\varepsilon}/\mathrm{Re}\tilde{\varepsilon}) \ll 1/\ln\ (2/\varkappa_0 a)$.

Similar calculations can be made for perturbation U, which is coordinate-dependent.

Thus, resonance scattering of radio waves in models of trails of rapidly moving bodies in the ionosphere depends significantly on trail symmetry, its electron concentration, and the polarization of the scattered radio waves. A quantitative evaluation of the resonance scattering cross section reveals that when conditions are favorable, the scattering cross section can exceed the effective trail surface by a factor of several score. /80

REFERENCES

1. Sayasov, Yu., Zhizhimov, L. A., Radiotekhnika i elektronika, No. 499, 1963.

2. Van-de-Huest, H. C., Rasseyaniye sveta malymi chastitsami [Scattering of Light by Small Particles], IL, 1961.

3. Morse, M., Feshbach, G., Metody teoreticheskoy fiziki [Methods of Theoretical Physics], Vol. II, IL, 1960.